

JEE MAIN 2026

Sample Paper – 8

Time Allowed: 3 hours

Maximum Marks: 300

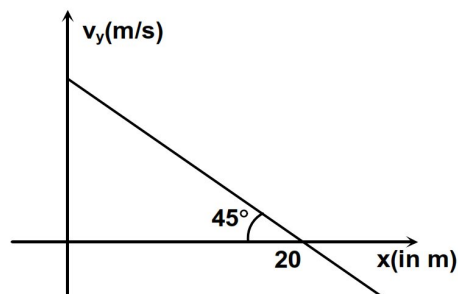
General Instructions:

1. The test consists of total **75 questions**.
2. Each subject (**PCM**) has **25 questions**.
3. Each subject divided into two sections. **Section A** consists of 20 multiple-choice questions & **Section B** consists of 5 numerical value-type questions.
4. **Marking Scheme:**
 - **Section A (MCQs):** +4 marks for each correct answer, –1 mark for each incorrect answer, 0 marks for unattempted.
 - **Section B (Numerical):** +4 marks for each correct answer, 0 marks for incorrect or unattempted.
5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.

SECTION – A (One Options Correct Type)

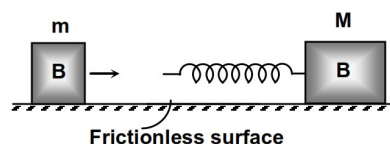
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A ball is projected from ground making an angle θ with the horizontal (x-direction). The vertical component of its velocity (v_y) changes with its x-coordinate according to the graph as shown in the figure. The angle of projection θ with horizontal is (Take $g = 10 \text{ m/s}^2$)



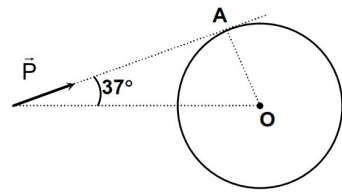
- (A) $\tan^{-1}(4)$ (B) $\tan^{-1}(2)$
(C) $\tan^{-1}\left(\frac{1}{4}\right)$ (D) $\tan^{-1}\left(\frac{1}{2}\right)$

2. Two blocks of mass m and M are lying on a smooth table. A spring is attached with the block of mass M . Block of mass m is given a velocity towards the other block. The value of $\frac{M}{m}$ for which the kinetic energy of the system will never fall below one fourth of the initial kinetic energy is



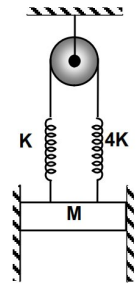
- (A) $\frac{M}{m} \leq 7$ (B) $\frac{M}{m} \leq 3$
(C) $\frac{M}{m} \leq 4$ (D) $\frac{M}{m} \leq 5$
3. A satellite is moving in a low circular orbit about a planet of mass M and radius R . The radius of the orbit can be taken to be R itself. Then the minimum increase in the speed required so that the satellite could escape from the gravitation pull of planet is
- (A) $\sqrt{\frac{2GM}{R}}$ (B) $\sqrt{\frac{GM}{2R}}$
(C) $\sqrt{\frac{GM}{R}}$ (D) $\sqrt{\frac{GM}{R}}(\sqrt{2} - 1)$
4. Two charged particle, having same kinetic energy are allowed to pass through a uniform magnetic field perpendicular to the direction of motion. If the ratio of radii of their circular paths is $6 : 5$ and their respective charges ratio is $1 : 2$. Then the ratio of their masses will be
- (A) $8 : 25$ (B) $9 : 25$
(C) $25 : 7$ (D) $6 : 25$

5. A short electric dipole of dipole moment P is placed near a neutral conducting sphere of radius R as shown in the figure. The electric potential at point A on the surface of sphere.

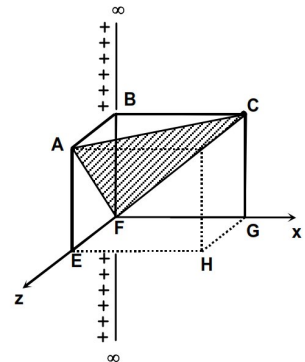


$$\left[K = \frac{1}{4\pi\epsilon_0} \right]$$

- (A) $\frac{30KP}{125R^2}$ (B) $\frac{125KP}{30R^2}$
 (C) $\frac{36KP}{125R^2}$ (D) $\frac{125KP}{36R^2}$
6. A block of mass M is suspended with the help of two light springs and light inextensible string that passes over an ideal pulley. Force constant of the spring are K and $4K$ respectively as shown. (walls are frictionless). The time period for small oscillations is

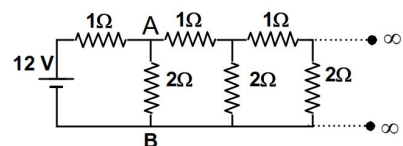


- (A) $\pi\sqrt{\frac{5M}{K}}$ (B) $\frac{\pi}{2}\sqrt{\frac{5M}{K}}$
 (C) $\pi\sqrt{\frac{M}{5K}}$ (D) $\pi\sqrt{\frac{2M}{5K}}$
7. A long straight wire having linear charge density λ passing through the edge of a cube of side length 'a' as shown in the figure. The electric flux through the triangular plane AFC is



- (A) zero (B) $\frac{\lambda a}{4\epsilon_0}$
 (C) $\frac{\lambda a}{2\epsilon_0}$ (D) $\frac{\lambda a}{8\epsilon_0}$

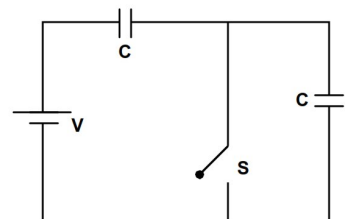
8. Figure shows an infinite ladder network of resistances. The current that passes through the branch AB.



- (A) 6A (B) 2A
 (C) 3A (D) 4A
9. If M is the mass of water rises in a capillary tube of radius 'r' then the mass of water which will rise in a capillary tube radius '4r' is

- (A) M (B) $4M$
 (C) $\frac{M}{2}$ (D) $2M$

10. In the circuit shown, find heat dissipated in the circuit after switch is closed (Initially switch is open)



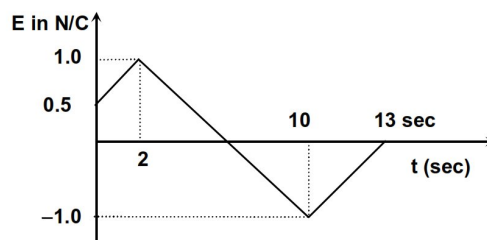
(A) $\frac{CV^2}{3}$

(B) $\frac{CV^2}{6}$

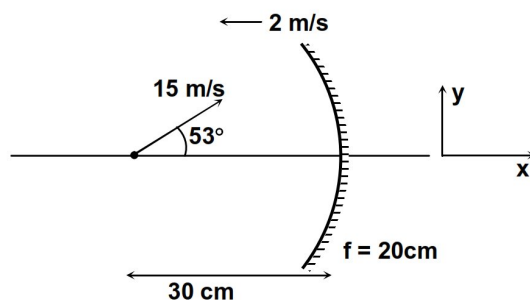
(C) $\frac{CV^2}{12}$

(D) $\frac{CV^2}{4}$

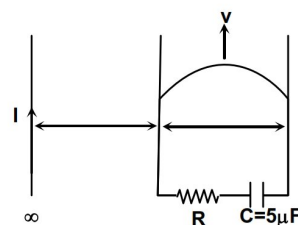
11. The electric field through an area of 2m^2 varies with time as shown in the graph. The greatest displacement current through the area is at
 (A) $t = 1$ sec
 (B) $t = 4$ sec
 (C) $t = 8$ sec
 (D) $t = 12$ sec



12. The velocity (in m/s) of image in a situation as shown in the figure is
 (A) $24\hat{i} - 46\hat{j}$
 (B) $-46\hat{i} - 24\hat{j}$
 (C) $46\hat{i} + 24\hat{j}$
 (D) $46\hat{i} - 24\hat{j}$

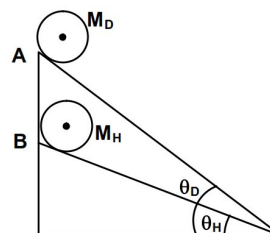


13. Two ends of a semicircular rod one at a distance 1 cm and 2 cm from a fixed long straight wire carrying current I as shown in the figure. The rod is made to move with constant speed of v on a conducting frame. At $t = 0$ capacitor is uncharged. The maximum change on the capacitor $C = 5\mu\text{F}$ is (Take $\frac{\mu_0}{4\pi} = 10^{-7}$ S.I. unit,



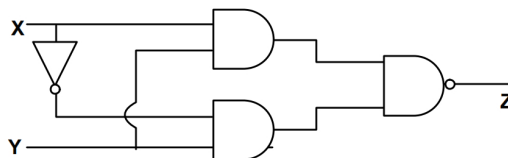
$$Iv = \frac{1}{\ln 2} \text{ S.I. unit}$$

- (A) $1 \times 10^{-12} \text{ C}$
 (B) $2 \times 10^{-12} \text{ C}$
 (C) $3 \times 10^{-12} \text{ C}$
 (D) $4 \times 10^{-12} \text{ C}$
14. A uniform disc of mass M_D and a uniform hollow sphere of mass M_H are placed at points A and B of two inclines, respectively. If they roll on the incline without slipping such that their acceleration are the same, then the ratio $\frac{\sin \theta_D}{\sin \theta_H}$
 (A) $\frac{5}{9}$
 (B) $\frac{9}{5}$
 (C) $\frac{9}{10}$
 (D) $\frac{8}{5}$



15. n moles of an ideal gas with constant volume heat capacity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is
 (A) $\frac{nR}{C_V - nR}$
 (B) $\frac{nR}{C_V + nR}$
 (C) $\frac{4nR}{C_V + nR}$
 (D) $\frac{4nR}{C_V - nR}$

16. Truth table for the given circuit is



(A)

X	Y	Z
1	0	1
0	0	1
1	1	1
0	1	1

(C)

X	Y	Z
1	0	1
0	0	1
1	1	1
0	1	0

(B)

X	Y	Z
1	0	0
0	0	0
1	1	0
0	1	1

(D)

X	Y	Z
1	0	0
0	0	1
1	1	1
0	1	1

17. In a young's Double slit experiment, D equals the distance of screen from slit and d is the separation between the slit. The distance of the nearest point to the central maxima where intensity is twice as that due to single slit is equal to ($D \gg d$)

(A) $\frac{D\lambda}{2d}$

(B) $\frac{D\lambda}{4d}$

(C) $\frac{D\lambda}{3d}$

(D) $\frac{2D\lambda}{d}$

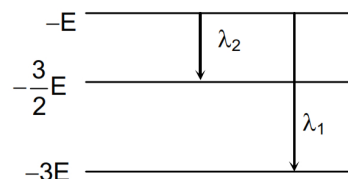
18. Figure shows energy levels of a molecule, the ratio of the wavelengths $\frac{\lambda_1}{\lambda_2}$ for transitions shown is

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) 1

(D) 2



19. The external and internal diameters of a hollow cylinder are measured to be (5.23 ± 0.01) cm and (4.89 ± 0.01) cm. The thickness of the wall of the cylinder is

(A) (0.17 ± 0.02) cm

(B) (0.17 ± 0.01) cm

(C) (0.34 ± 0.01) cm

(D) (0.34 ± 0.02) cm

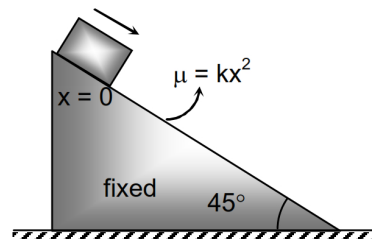
20. A block is released from rest on a rough inclined plane with coefficient of friction varying as $\mu = kx^2$; where k is constant. The maximum velocity of block for

(A) $x = \frac{1}{\sqrt{k}}$

(B) $x = \frac{2}{\sqrt{k}}$

(C) $x = \frac{4}{\sqrt{k}}$

(D) $x = \frac{3}{\sqrt{k}}$

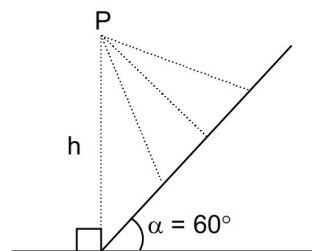


SECTION – B

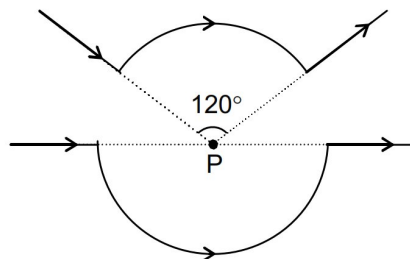
(Numerical Answer Type)

This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

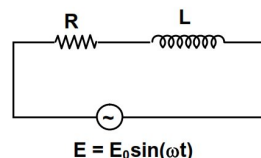
21. A point P lies vertically above the lowest point of an inclined plane of inclination $\alpha = 60^\circ$ at a height $h = 30$ m. Point P is joined to the plane at number of points by smooth wires, running in all possible directions. Small bodies in the shape of beads are released from P along the wire simultaneously. Find the least time (in sec) taken by bead to reach the inclined plane. (Take $g = 10 \text{ m/s}^2$)



22. Figure shows two current segments. In the upper segment, an arc of radius 4 cm subtends an angle of 120° with centre P. The lower segment includes a large semicircle of radius 5 cm also with centre P. If $I = 0.2$ amp in both. If the net magnetic field at point P due to these current segments is $\frac{\pi x}{30} \mu\text{T}$, then find the value of x.



23. A charged particle of $1 \mu\text{C}$ and having mass 2 gm is projected with speed of 10 m/s from a point where electric potential is $4 \times 10^5 \text{ V}$ in the direction of decreasing electric potential. The electric potential at a point where its speed becomes $10\sqrt{2} \text{ m/s}$ is $n \times 10^5$ volts. Find the value of n?
24. Figure show a series LR circuit, when instantaneous voltage across source is maximum. The voltage across inductor is 3V and that across resistor is 4V. If resistance is 2Ω , the reactance of inductor is $\sqrt{n} \Omega$, find the value of n.



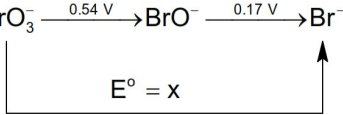
25. Two travelling wave produces standing wave represented by equation

$$y = 2(\text{mm}) \cos\left(\frac{\pi}{4} \text{cm}^{-1}\right) x \sin(78.5 \text{s}^{-1})t$$
 The node closet to the origin in the region $x > 0$ will be at $x = \dots\dots\dots \text{cm}$



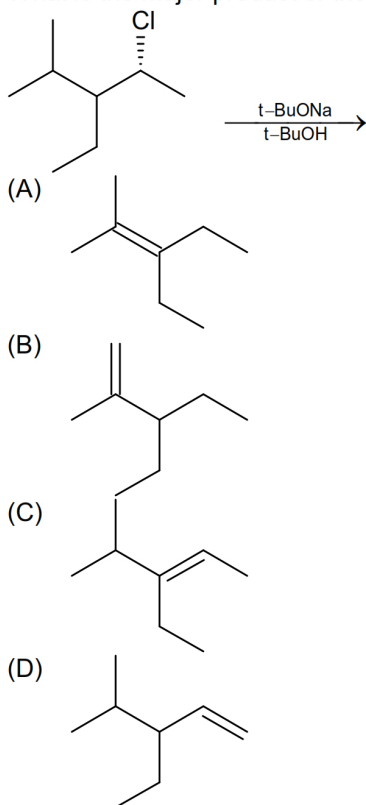
SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

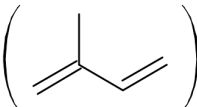
26. Decomposition of $\text{Cl}_2\text{O}_7(\text{g})$ proceeds by zero order kinetics as:
 $\text{Cl}_2\text{O}_7(\text{g}) \longrightarrow 2\text{ClO}_2(\text{g}) + \frac{3}{2}\text{O}_2(\text{g})$. If $P_0 = 600$ mm Hg and $-\frac{d[\text{Cl}_2\text{O}_7]}{dt} = 50$ mm Hg. Then rate of production of O_2 and half-life period of reaction are
 (A) 50 mm Hg, 10 sec (B) 60 mm Hg, 6 sec
 (C) 75 mm Hg, 6 sec (D) 75 mm Hg, 10 sec
27. $\text{BrO}_3^- \xrightarrow{0.54 \text{ V}} \text{BrO}^- \xrightarrow{0.17 \text{ V}} \text{Br}^-$

 The value of x is
 (A) - 0.417 V (B) 0.717 V
 (C) 0.37 V (D) 0.417 V
28. The decreasing order of the rate of nitration of the following compounds is:
 I. Benzene II. C_6D_6
 III. Nitrobenzene IV. Chlorobenzene
 (A) I > II > III > IV (B) I > II > IV > III
 (C) I = II > IV > III (D) I = II > III > IV
29. Choose the incorrect statement
 (A) $\text{NF}_3 < \text{NCl}_3 < \text{NBr}_3$ (Lewis base strength)
 (B) In hydrolysis of SbCl_3 the addition of excess of HCl suppresses the hydrolysis by shifting the equilibrium to the left
 (C) NF_3 does not undergo hydrolysis
 (D) Thermal stability of $\text{PCl}_5 > \text{PCl}_3$
30. Equal volume each of two sols of AgI, one obtained by adding AgNO_3 to slight excess of KI and another obtained by adding KI to slight excess of AgNO_3 are mixed together. Then:
 (A) The sols will coagulate each other mutually
 (B) A true solution will be obtained
 (C) The two sols will stabilize each other
 (D) The sol will stabilize each other
31. In H-like species, an electron jumps from an orbital having two radial and two angular nodes to the orbital having same sign of wave function in all direction, at any distance. If energy of the emitted photon is 326.4 eV, the species is:
 (A) H (B) He^+
 (C) Li^{+2} (D) B^{+4}



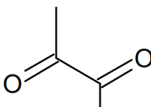
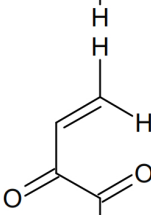
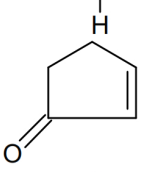
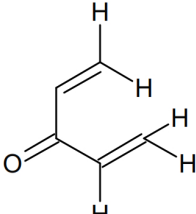
32. 2 moles of ideal gas undergoes following process:
 (1) A reversible isobaric expansion from 1 atm, 20 litre to 1 atm 40 litres. (2) A reversible isochoric change to 0.5 atm. (3) A reversible isothermal compression from 0.5 atm to 1.0 atm at 27°C. The total work in the process is
 [Given: 1 litre atm = 100 J]
 (A) 19.89 J (B) 1415 J
 (C) 20.11 J (D) 22.22 J
33. 10 ml of 0.2 M solution of $K_xH(C_2O_4)_y$ requires 8 mL of 0.2 M acidified $KMnO_4$ solution. Then the value of x is?
 (A) 1 (B) 2
 (C) 3 (D) 4
34. Which of the following statement is correct with respect to bond angle
 (A) The $F-\hat{K}r-F$ angle in KrF_4 is 90° .
 (B) The $F-\hat{S}-F$ angle in SF_2 is more than $109^\circ 28'$.
 (C) The $H-\hat{N}-N$ angle in N_2H_2 is approximately 180° .
 (D) The $Cl-\hat{N}-O$ angle in $NOCl$ is more than 120° .
35. What is the major product of the reaction?



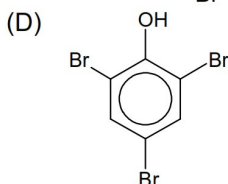
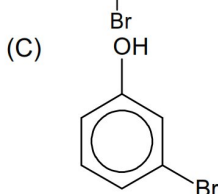
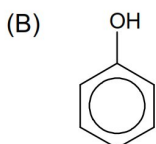
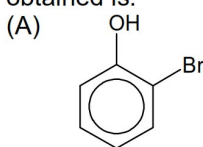
36. In the reaction,
 $3\text{Br}_2 + 6\text{CO}_3^{--} + 3\text{H}_2\text{O} \longrightarrow 5\text{Br}^- + \text{BrO}_3^- + 6\text{HCO}_3^-$
 (A) Bromine undergoes disproportionation
 (B) Equivalent weight of CO_3^{--} is $\frac{10M}{5}$
 (C) Equivalent weight of Br_2 is $\frac{3M}{5}$
 (D) CO_3^{--} is reducing agent
37. A gaseous equilibrium $\text{A(g)} + \text{B(g)} \rightleftharpoons 2\text{C(g)}$, rate constant forward and backward reactions are 6.4×10^{-4} and 1.6×10^{-4} respectively, starting with initially 2 moles each of A and B in a two litres contains equilibrium concentration of C is,
 (A) 1.2 (B) 1.66
 (C) 1.00 (D) 1.42

38. Natural rubber is the polymer of isoprene 

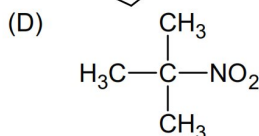
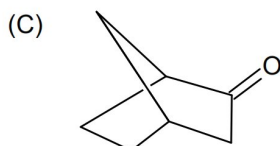
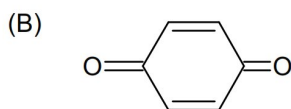
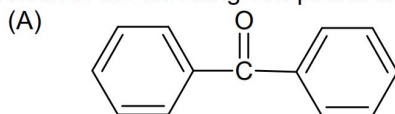
Natural rubber $\xrightarrow[2. \text{Zn} + \text{H}_2\text{O}]{1. \text{O}_3} [\text{X}] \xrightarrow[2. \text{H}^+, \Delta]{1. \text{OH}^- (\text{aq})} [\text{Y}]$. The compound [Y] is

- (A) 
- (B) 
- (C) 
- (D) 

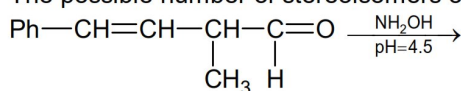
39. Non-volatile solute 'A' dimerises in a solvent as $2A \rightleftharpoons A_2$. If m is the molarity. Then equilibrium constant for reaction is
- (A) $\frac{2K_b(K_b m + \Delta T_b)}{(2\Delta T_b - K_b m)^2}$ (B) $\frac{K_b(K_b m - \Delta T_b)}{(2\Delta T_b - K_b m)^2}$
- (C) $\frac{K_b(K_b m - \Delta T_b)}{(2\Delta T_b - K_b m)^2}$ (D) $\frac{K_b(K_b m - \Delta T_b)}{(2\Delta T_b + K_b m)^2}$
40. The basic ionisation constant for hydrazine, N_2H_4 is 9.6×10^{-7} . What would be the percent hydrolysis of $0.1 N_2H_5Cl$?
- (A) 0.016% (B) 3.2%
(C) 1.6% (D) 0.032%
41. The number of chiral carbons in the product formed when glucose reacts with HCN is
- (A) 5 (B) 4
(C) 6 (D) 3
42. On reaction of cis-2-phenyl-1-bromocyclopentane with alc. KOH produces a compound (X). (X) on reaction with dil. H_2SO_4 produces (Y). Which of the following statement is incorrect for (X) and (Y)?
- (A) Product (Y) is a 3° ROH, with a chiral carbon.
(B) IUPAC name of (X) is 1-phenylcyclopentene.
(C) No rearrangement occurs when product (X) is made to react with dil. H_2SO_4 .
(D) (X) and (Y) both are optically inactive.
43. When phenol is heated with an aqueous solution of mixture of KBr and $KBrO_3$; the major product obtained is:



44. Which of the following compound exhibits tautomerism?



45. The possible number of stereoisomers of the product of following reaction would be:



- (A) 2
(C) 6

- (B) 4
(D) 8

SECTION – B

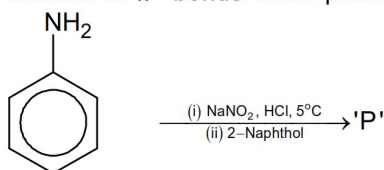
(Numerical Answer Type)

This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

46. The number of ions having same value of spin only moment (μ_s) in Fe^{2+} , Mn^{2+} , Cr^{3+} and Ni^{2+} is....

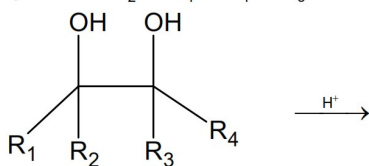
47. The number of species having higher IE_1 than Ca from the following is.....
Ga, Ge, Br, Se, Kr, As, K

48. Number of π -bonds in the product 'P' in the following reaction is ____



49. An optically pure organic compound has specific rotation of $+40^\circ\text{C}$. The optical purity of the sample that exhibits specific rotation of $+32^\circ$ is _____ %

50. If total number of possible product in the following molecule are 'x' when $\text{R}_1 \neq \text{R}_2 \neq \text{R}_3 \neq \text{R}_4$ and 'y' when $\text{R}_2 = \text{R}_4 \neq \text{R}_1 \neq \text{R}_3$ then the value of 'x + y' is (R_1 , R_2 , R_3 and R_4 are alkyl group).



SECTION – A (One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

51. Let M and N are square matrices of same order satisfying $MN = M$ and $NM = N$, then $(M^{2024} + N^{2024})^{2025}$ is equal to
 (A) $M + N$ (B) $2025(M + N)$
 (C) $2^{2024}(M + N)$ (D) $2^{2025}(M + N)$
52. If $\left(x - 2 + \frac{1}{x}\right)^{30} = a_0x^{30} + a_1x^{29} + \dots + a_{29}x + a_{30}x^{-1} + \dots + a_{60}x^{-30}$ and $k = a_0 + a_1 + a_2 + \dots + a_{60}$ if $k - a_{30} = -{}^nC_r$, then $n + r$ is equal to
 (A) 50 (B) 80
 (C) 90 (D) 110
53. Four digit numbers are formed using the digits from the set $\{0, 1, 2, 3, 4, 5\}$ repetition of digits is allowed then :
 Statement (S_1) : The number of such numbers formed that are odd is 480.
 Statement (S_2) : The number of such numbers formed such that it contains exactly three different digits is 360.
 (A) S_1, S_2 are true (B) S_1, S_2 are false
 (C) S_1 is true and S_2 is false (D) S_1 is false and S_2 is true
54. The number of points in the complex plane that satisfy the condition $|z - 2| = 2$, $z(1 - i) + \bar{z}(1 + i) = 4$ where $i = \sqrt{-1}$, is
 (A) 0 (B) 1
 (C) 2 (D) more than 2
55. Let $T(\theta) = \cos^2(30^\circ - \theta^\circ) - \cos(30^\circ - \theta^\circ) \cos(30^\circ + \theta^\circ) + \cos^2(30^\circ + \theta^\circ)$, then value of $4 \sum_{\theta=1}^{30} \theta T(\theta) =$
 (A) 1395 (B) 1295
 (C) 995 (D) 895
56. The number of three term increasing geometrical progressions comprising distinct natural numbers less than or equal to 100, with common ratio as a natural number is
 (A) 106 (B) 53
 (C) 47 (D) 104
57. If a and b are chosen randomly by throwing a pair of fair cubical dice, then the probability that $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{2/x} = 6$ equals
 (A) $\frac{1}{9}$ (B) $\frac{2}{9}$
 (C) $\frac{3}{9}$ (D) $\frac{4}{9}$

58. Let $f(x)$ be a cubic polynomial on $\mathbb{R}$ which increases in interval $(-\infty, 0)$ and in $(1, \infty)$ and decreases in interval $(0, 1)$. If $f'(2) = 6$ and $f(2) = 2$, then the value of $\tan^{-1}(f(1)) + \tan^{-1}\left(f\left(\frac{3}{2}\right)\right) + \tan^{-1}(f(0))$ is equal to
- (A) $\tan^{-1}2$ (B) $\cot^{-1}2$
(C) $-\tan^{-1}2$ (D) $-\cot^{-1}2$
59. If the logarithms of two distinct positive numbers are equal to the corresponding numbers then base of logarithms belongs to in interval
- (A) $(0, 1)$ (B) $(1, e)$
(C) (e, e^e) (D) $(1, e^{1/e})$
60. Let $y = y(x)$ satisfy the differential equation $\left(2xy + x^2y + \frac{y^3}{3}\right)dx + (x^2 + y^2)dy = 0$. If $y(1) = 1$ and $(y(0))^3 = ke$, $k \in \mathbb{N}$ then k is
- (A) 3 (B) 4
(C) 1 (D) 2
61. A strictly increasing continuous function $f(x)$ intersects with its inverse $f^{-1}(x)$ at $x = \alpha$ and $x = \beta$, $\int_{\alpha}^{\beta} (f(x) + f^{-1}(x))dx = 13$, where $\alpha, \beta \in \mathbb{N}$ then the value of $|\alpha\beta|$ equals
- (A) 25 (B) 36
(C) 42 (D) 56
62. Given lines $\frac{x}{a} + \frac{y}{b} = 1$ and $ax + by = 1$ are two variable lines, 'a' and 'b' being the parameters connected by the relation $a^2 + b^2 = ab$. The locus of the point of intersection has the equation
- (A) $x^2 + y^2 + xy - 1 = 0$ (B) $x^2 + y^2 - xy + 1 = 0$
(C) $x^2 + y^2 + xy + 1 = 0$ (D) $x^2 + y^2 - xy - 1 = 0$
63. Let $H_n : \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1$, $n \in \mathbb{N}$. Let k be the smallest even value of n such that the eccentricity of H_k is rational number. If ' ℓ ' is length of the latus rectum of H_k , then 21ℓ is equal to
- (A) 101 (B) 204
(C) 102 (D) 306
64. In a parabola $y^2 = 4ax$, two points P and Q are taken such that the tangents drawn to parabola at these points meet at directrix in R. Focus of locus of circumcentre of triangle PQR will be
- (A) $\left(\frac{a}{2}, 0\right)$ (B) $(a, 0)$
(C) $\left(\frac{3a}{2}, 0\right)$ (D) $\left(\frac{5a}{2}, 0\right)$
65. If circle is inscribed in ellipse $x^2 + 4y^2 = 4$, then range of radius of circle is
- (A) $[0, 1)$ (B) $(0, 1]$
(C) $\left[\frac{1}{2}, 1\right]$ (D) $\left[\frac{1}{2}, 1\right]$

66. Let $\vec{a}, \vec{b}, \vec{c}$ are three vectors having magnitudes 1, 2, 3 respectively satisfy the relation $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = 6$. If $\vec{d}$ is a unit vector coplanar with $\vec{b}$ and $\vec{c}$ such that $\vec{b} \cdot \vec{d} = 1$ then the value of $|(\vec{a} \times \vec{c}) \cdot \vec{d}|^2 + |(\vec{a} \times \vec{c}) \times \vec{d}|^2$ is
 (A) 9 (B) 3
 (C) 27 (D) $\frac{9}{2}$
67. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors satisfying $\vec{a} = \vec{b} \times \vec{c} + 2\vec{b}$ where $|\vec{b}| = |\vec{c}| = 2$ and $|\vec{a}| \leq 4$. The sum of possible value(s) of $|2\vec{a} + \vec{b} + \vec{c}|$ is
 (A) 8 (B) 12
 (C) 20 (D) 32
68. If the equation of the plane passing through the point $(-1, 2, 0)$ and parallel to the lines $\frac{x}{3} = \frac{y+1}{0} = \frac{z-2}{-1}$ and $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-1}$ is $ax + by + cz = 1$, then the value of $a + b + c$ is
 (A) 3 (B) 4
 (C) 5 (D) 10
69. Let $A = \{1, a_1, a_2, \dots, a_{18}, 77\}$ be set of integers with $1 < a_1 < a_2 < \dots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then the value of $a_1 + a_2 + \dots + a_{18}$ is equal to
 (A) 600 (B) 702
 (C) 800 (D) 200
70. Consider the data on X taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0, {}^nC_1, \dots, {}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to
 (A) 15 (B) 8
 (C) 4 (D) 6

SECTION – B

(Numerical Answer Type)

This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

71. The number of integral values of k for which line $3x + 4y - k = 0$, lies between the circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 12y + 113 = 0$, without cutting a chord on either of circle is equal to
72. Let E_1, E_2, E_3 be three independent events associated with a random experiment such that $3P(\overline{E_1} \cap \overline{E_2} \cap \overline{E_3}) = P(\overline{E_1} \cap E_2 \cap \overline{E_3}) = 9P(\overline{E_1} \cap \overline{E_2} \cap E_3) = 3 - 3P(E_1 \cup E_2 \cup E_3)$, where $P(E_1), P(E_2), P(E_3) \neq 1$ and $P(A)$ denotes probability of event A. If absolute value of $\begin{vmatrix} P(E_1) & P(E_2) & P(E_3) \\ P(E_2) & P(E_3) & P(E_1) \\ P(E_3) & P(E_1) & P(E_2) \end{vmatrix} = \frac{a}{b}$ where $a, b \in \mathbb{N}$, then least value of $a + b$ is

73. If the length of the perpendicular drawn from the point $P(a, 4, 2)$, $a > 0$ on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $Q(\alpha_1, \alpha_2, \alpha_3)$ is the image of point P on this line then $a + \sum_{i=1}^3 \alpha_i$ is equal to
74. Let 'f' be a function defined on the interval $(0, 2\pi]$ such that $\int_0^x (f'(t) - \sin 2t) dt = \int_x^0 f(t) \tan t dt$ and $f(0) = 1$, if the maximum value of $f(x)$ is m then $8m$ is equal to
75. The eccentricity of ellipse $3x^2 + 4y^2 = 12$ is changed at the rate $0.1/\text{sec}$. The time in sec. such that ellipse becomes auxiliary circle is

SOLUTIONS

Physics

PART – A

SECTION – A

1.

B

Sol.

$$v_y = u_y - gt$$

...(i)

$$x = u_x t$$

...(ii)

$$v_y = u_y - g \frac{x}{u_x}$$

$$\Rightarrow \frac{g}{u_x} = \tan 45^\circ \Rightarrow u_x = 10 \text{ m/s}$$

$$\text{Also, Range } R = \frac{2u_x u_y}{g}$$

$$40 = \frac{2 \times 10 \times u_y}{g} \Rightarrow u_y = 20 \text{ m/s}$$

$$\tan \theta = \frac{u_y}{u_x} = 2$$

$$\Rightarrow \theta = \tan^{-1}(2)$$

2.

B

Sol.

Maximum loss in kinetic energy

$$\Delta K.E_{\max} = \frac{1}{2} \frac{Mm}{(M+m)} u^2$$

$$\Delta K.E_{\max} \leq \frac{3}{4} \left(\frac{1}{2} m u^2 \right)$$

$$\frac{1}{2} \frac{Mm}{(M+m)} u^2 \leq \frac{3}{4} \left(\frac{1}{2} m u^2 \right)$$

$$\Rightarrow \frac{M}{m} \leq 3$$

3.

D

Sol. $-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0$

$$\frac{GMm}{R} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2GM}{R}}$$

$$\Delta v = (\sqrt{2} - 1)\sqrt{\frac{GM}{R}}$$

4. B

Sol. $R = \frac{\sqrt{2mk}}{Bq}$

$$m \propto R^2 q^2$$

$$\frac{m_1}{m_2} = \left(\frac{R_1 q_1}{R_2 q_2} \right)^2$$

$$\left(\frac{6}{5} \times \frac{1}{2} \right)^2 = \frac{9}{25}$$

5. C

Sol. $V_A = \frac{KP \cos 37^\circ}{\left(\frac{5R}{3} \right)^2} = \frac{9KP}{25R^2} \times \frac{4}{5}$

$$V_A = \frac{36KP}{125R^2}$$

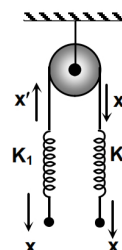
6. B

Sol. $F_R = 2K_2(x - x')$

$$F_R = \frac{4K_1 K_2 x}{(K_1 + K_2)}$$

$$K_2(x - x') = K_1(x + x')$$

$$x' = \frac{(K_2 - K_1)x}{(K_1 + K_2)}$$



7. B

Sol. $\phi_{AFC} = \phi_{\text{cube}} = \frac{\lambda a}{4\epsilon_0}$

8. C

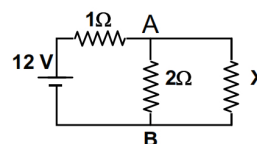
Sol. Let equivalent resistance across battery be x

$$\frac{2x}{2+x} + 1 = x$$

$$\Rightarrow x = 2 \Omega$$

$$V_{AB} = 6V$$

$$I_{AB} = 3A$$



9. B

Sol. $T2\pi r \cos \theta = Mg$

$$\Rightarrow M \propto r$$

$$\frac{M_1}{M_2} = \frac{r_1}{r_2} = \frac{r}{4r}$$

$$M_2 = 4M_1$$

$$M_2 = 4M$$

10. D

Sol. $U_i + W_B = U_f + \text{Heat}$

$$\frac{CV^2}{4} + \frac{CV^2}{2} = \frac{CV^2}{2} + \text{Heat}$$

$$\text{Heat} = \frac{CV^2}{4}$$

11. D

Sol. $I_D = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 (\text{slope})$

12. B

Sol. $\vec{v}_{OG} = 9\hat{i} + 12\hat{j}$, $\vec{v}_{MG} = -2\hat{i}$

For x-axis: $\vec{v}_{IM} = -m^2 \vec{v}_{OM}$

For y-axis: $\vec{v}_{I_y} = -m \vec{v}_{O_y}$

$$\vec{v}_{IG} = -46\hat{i} - 24\hat{j} \text{ m/s}$$

13. A

Sol. $e = \int \frac{\mu_0 I dx}{2\pi x} = \frac{\mu_0 I v}{2\pi} \ln 2$

$$q_{\max} = Ce$$

14. C

Sol. $a = \frac{g \sin \theta}{\left(1 + \frac{I_{cm}}{MR^2}\right)}$

$$\frac{\sin \theta_D}{\left(1 + \frac{1}{2}\right)} = \frac{\sin \theta_H}{\left(1 + \frac{2}{3}\right)}$$

$$\frac{2 \sin \theta_D}{3} = \frac{3 \sin \theta_H}{5}$$

$$\frac{\sin \theta_D}{\sin \theta_H} = \frac{9}{10}$$

15. B

Sol. $\frac{nR\Delta T}{n\left(R + \frac{C_V}{n}\right)\Delta T}$

$$\Rightarrow \frac{nR}{C_V + nR}$$

16. A

Sol. Conceptual

17. B

Sol. $2I_0 = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

$$\frac{1}{\sqrt{2}} = \cos\left(\frac{\phi}{2}\right)$$

$$\frac{\phi}{2} = \frac{\pi}{4}$$

$$\phi = \frac{\pi}{2}$$

$$\Rightarrow \Delta x = \frac{\lambda}{4}$$

$$\frac{yD}{D} = \frac{\lambda}{4}$$

$$y = \frac{\lambda D}{4d}$$

18. B

Sol. $\Delta E = \frac{hC}{\lambda}$

$$\frac{E}{2} = \frac{hC}{\lambda_2}$$

...(i)

$$2E = \frac{hC}{\lambda_1}$$

...(ii)

$$\frac{1}{4} = \frac{\lambda_1}{\lambda_2}$$

19. B

Sol. Conceptual

20. A

Sol. $-\int_0^x mg \cos \theta kx^2 dx + \int_0^x mg \sin \theta dx = \frac{1}{2}mv^2$

$$-mg \cos \theta \frac{kx^3}{3} + mg \sin \theta x = K.E.$$

For K.E. to be maximum

$$\frac{dK.E.}{dx} = -mg \cos \theta kx^2 + mg \sin \theta = 0$$

SECTION – B

21. 2

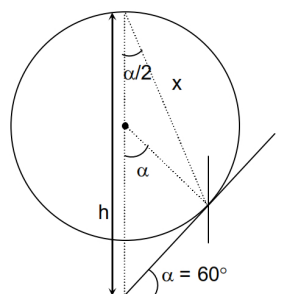
Sol. For time 't' all beads lie on a circle

$$x = \frac{h}{\sqrt{3}}$$

$$x = \frac{1}{2}g \cos\left(\frac{\alpha}{2}\right)t^2, \quad \frac{h}{\sqrt{3}} = \frac{1}{2}g \cos 30^\circ t^2$$

$$\frac{h}{\sqrt{3}} = \frac{1}{2}g \frac{\sqrt{3}}{2}t^2$$

$$t = 2 \text{ sec}$$



22. 2

Sol. Magnetic field due to upper loop $(B_1) = \frac{\mu_0(i)}{4\pi r_1} \left(\frac{2\pi}{3} \right) (-\hat{z})$

Magnetic field due to lower loop $(B_2) = (B_2) = \frac{\mu_0(i)}{4\pi r_2} \pi(+\hat{z})$

23. 3

Sol. $q[v_i - v_f] = k_f - k_i$

$$10^{-6} [4 \times 10^5 - v_f] = \frac{1}{2} \times 2 \times 10^{-3} [200 - 100]$$

$$4 \times 10^5 - v_f = 10^5$$

$$v_f = 3 \times 10^5 \text{ volts}$$

$$n = 3$$

24. 3

Sol. For instantaneous values

$$\text{Let } I = I_0 \sin(\omega t - \phi)$$

$$V_R = IR = I_0 R \sin(\omega t - \phi)$$

$$\Rightarrow I_0 \sin(\omega t - \phi) = 2$$

$$\frac{E_0}{Z} \sin(\omega t - \phi) = 2$$

$$\cos \phi = \frac{R}{Z} = \frac{2Z}{E_0}$$

$$\Rightarrow x_L = \sqrt{3} \Omega$$

$$V_L + V_R = E = 7$$

$$\Rightarrow E = 7V$$

$$E = E_0 \sin(\omega t)$$

$$7 = 7 \sin(\omega t)$$

$$\sin \omega t = 1$$

$$\omega t = \frac{\pi}{2}$$

25. 2

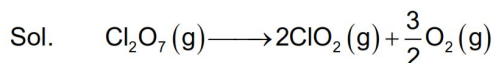
Sol. $\frac{2\pi}{\lambda} = \frac{\pi}{4}$

$$\lambda = 8 \text{ cm}$$

$$1^{\text{st}} \text{ Node: } \frac{\lambda}{4} = 2 \text{ cm}$$

SECTION – A

26. C



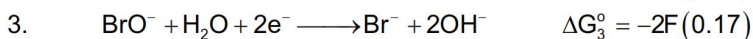
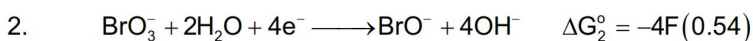
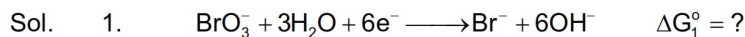
$$\text{Rate} = -\frac{d[\text{Cl}_2\text{O}_7]}{dt} = \frac{1}{2} \frac{d[\text{ClO}_2]}{dt} = \frac{3}{2} \frac{d[\text{O}_2]}{dt}$$

$$\frac{d[\text{O}_2]}{dt} = 50 \times \frac{3}{2} = 75 \text{ mm Hg}$$

$$\text{Rate } k[\text{Cl}_2\text{O}_7]^0 = k = 50$$

$$t_{1/2} = \frac{P_0}{2k} = \frac{600}{2 \times 50} = 6 \text{ sec}$$

27. D



$$\text{Eq. (2) + (3)} = 1, = \Delta G_1^\circ = \Delta G_2^\circ + \Delta G_3^\circ = -F(4 \times 0.54 + 2 \times 0.17) = -2.5 \text{ VF}$$

$$E = \frac{\Delta G_1^\circ}{nF} = -\frac{(2.5)\text{VF}}{6F} = 0.41 \text{ V}$$

28. C

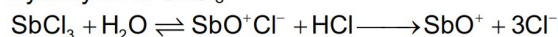
Sol. Rate is faster when the substituent activates the ring (+I or/and +R, o/p) and the rate is slower when the substituent deactivates the ring (– I, R; m). Halogen deactivates the ring (– I, +R, – I > +R) but the orientation is o/p.

Rate of $\text{C}_6\text{H}_6 = \text{C}_6\text{D}_6$, since no kinetic isotope effect is observed when H is replaced by D.

Hence, the order is as given in option (C).

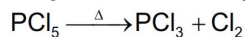
29. D

Sol. Hydrolysis of SbCl_3



NF_3 does not undergo hydrolysis, due to high N – F bond strength.

Pentahalides are thermally less stable than the corresponding trihalides, e.g. thermal stability of PCl_3 is less than PCl_5

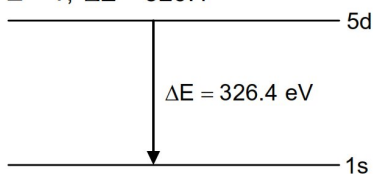


30. A

Sol. The sols obtained in the two cases will be oppositely charged so coagulate each other.

31. D

Sol. $Z = ?$; $\Delta E = 326.4$



$$= 13.6 Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \text{eV}$$

$$\Rightarrow 326.4 = 13.6 \times Z^2 \left[\frac{1}{(1)^2} - \frac{1}{(5)^2} \right] \Rightarrow Z = 5$$

32. B

Sol. $W_{\text{total}} = W_{AB} + W_{BC} + W_{CA}$

$$= 1(40 - 20) + 0 + \left(-nRT \ln \frac{20}{40} \right)$$

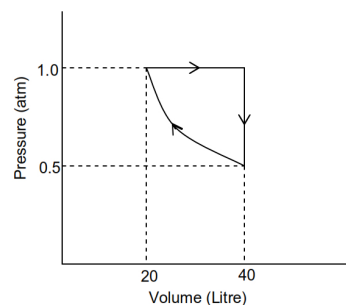
$$= -20 \text{ litre atm} + 2 \times 2.303 \times 0.0821 \log 2 \times 300$$

$$= -20 \text{ litre atm} + 34.15 \text{ litre atm}$$

$$= 14.15 \text{ litre atm}$$

$$= 14.15 \times 100 \text{ J}$$

$$= 1415 \text{ J}$$



33. C

Sol. meq. of $\text{KMnO}_4 = \text{meq. of } \text{K}_x\text{H}(\text{C}_2\text{O}_4)_y$

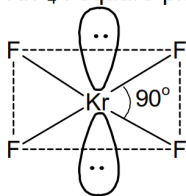
$$0.2 \times 5 \times 8 = 10 \times 0.2 \times \text{n-factor}$$

$$\text{n-factor} = 4 \Rightarrow x = 3$$

$$\text{K}_3\text{H}(\text{C}_2\text{O}_4)_2$$

34. A

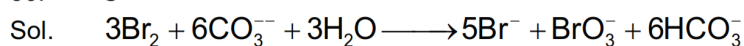
Sol. KrF_4 : square planar.



35. D

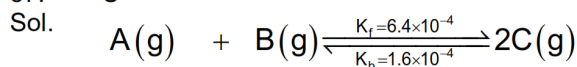
Sol. With bulky base, Hofmann's elimination occur to give the major elimination product.

36. C



$$\text{EW}(\text{Br}_2) = \frac{M}{2} + \frac{M}{10} = \frac{3M}{5}$$

37. C

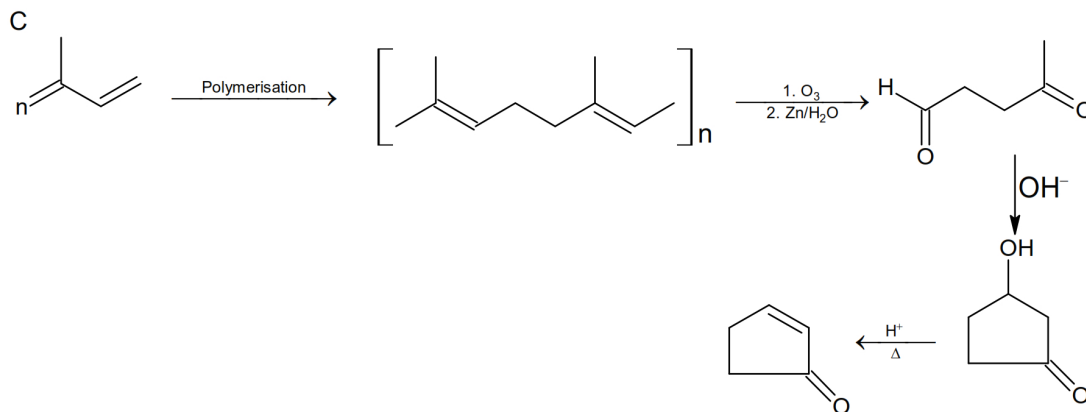


2	2	0	initially
$2 - x/2$	$2 - x/2$	$2x/2$	equilibrium

$$K_c = \frac{[\text{C}]^2}{[\text{A}][\text{B}]} = \frac{4x^2 \times 4}{4(2-x)^2} = \frac{6.4 \times 10^{-4}}{1.6 \times 10^{-4}}$$

$$\begin{aligned} \text{or } \frac{4x}{2-x} &= 4 & \text{or } 4x &= 8-4x \\ \text{or } 8x &= 8, & x &= 1 \\ \text{Hence, } \frac{2x}{2} &= \frac{2 \times 1}{2} = 1 \end{aligned}$$

38.
Sol.



39.
Sol.

C

$$2A \rightleftharpoons A_2$$

$$1-\alpha \quad \alpha/2$$

$$i = 1 - \frac{\alpha}{2} \quad \text{or} \quad \alpha = 2(1-i)$$

$$K_{eq} = \frac{m \left(\frac{\alpha}{2} \right)}{m^2 (1-\alpha)^2} = \frac{1-i}{m(2i-1)^2} \quad \text{where } i = \frac{\Delta T_b}{(\Delta T_b)} = \frac{\Delta T_b}{K_b m}$$

$$K_{eq} = \frac{1 - \Delta T_b / K_b m}{m[(2\Delta T_b / K_b m) - 1]^2} = \frac{K_b (K_b m - \Delta T_b)}{(2\Delta T_b - K_b m)^2}$$

40.
Sol.

D

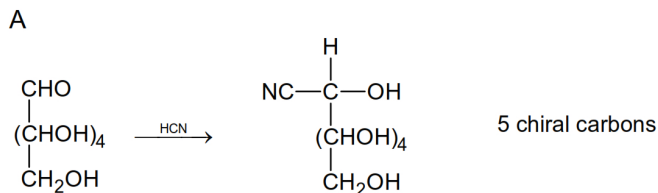
$$N_2H_5^+ + H_2O \rightleftharpoons N_2H_4 + H_3O^+$$

$$K_h = \frac{[N_2H_4][H_3O^+]}{[N_2H_5^+]} = \frac{k_w}{k_b} = \frac{10^{-14}}{9.6 \times 10^{-7}} = \frac{x^2}{0.1} = 1.04 \times 10^{-8}$$

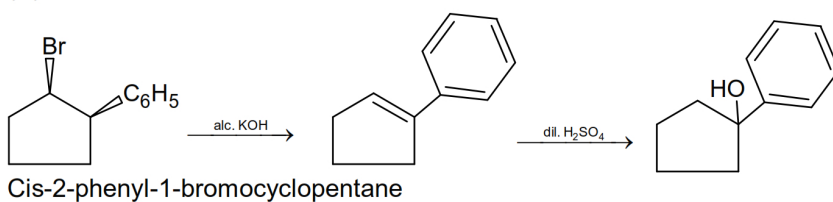
$$\therefore x = 3.2 \times 10^{-5}$$

$$\% \text{ hydrolysis} = \frac{3.2 \times 10^{-5}}{0.1} \times 100 = 0.032 \%$$

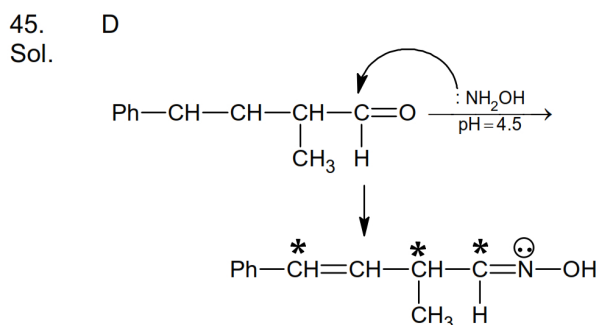
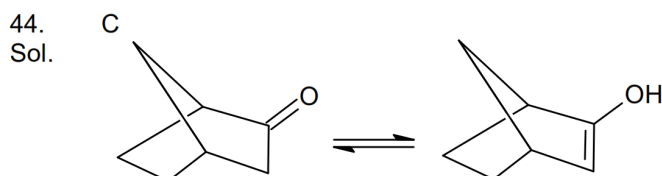
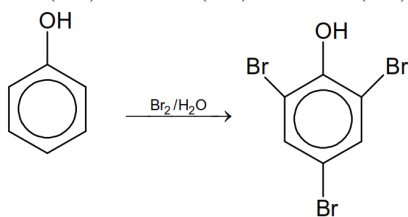
41.
Sol.



42. A
Sol. (A) is incorrect.



43. D
Sol. $\text{KBr(aq.)} + \text{KBrO}_3(\text{aq.}) \longrightarrow \text{Br}_2(\text{aq.})$



SECTION – B

46. 2
Sol. $\text{Fe}^{++} \equiv [\text{Ar}]_{18} 3d^6 \Rightarrow n = 4$
 $\text{Mn}^{++} \equiv [\text{Ar}]_{18} 3d^5 \Rightarrow n = 5$
 $\text{Cr}^{++} \equiv [\text{Ar}]_{18} 3d^4 \Rightarrow n = 4$
 $\text{Ni}^{++} \equiv [\text{Ar}]_{18} 3d^8 \Rightarrow n = 2$

Since, $\mu_s = \sqrt{n(n+2)} \text{ BM}$

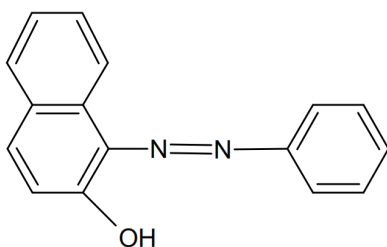
Fe^{++} and Cr^{++} will have same μ_s .

47. 5

Sol. The correct order in 4th period is
 $K < Ga < Ca < Ge < Se < As < Br < Kr$

48. 9

Sol. Product 'P' is

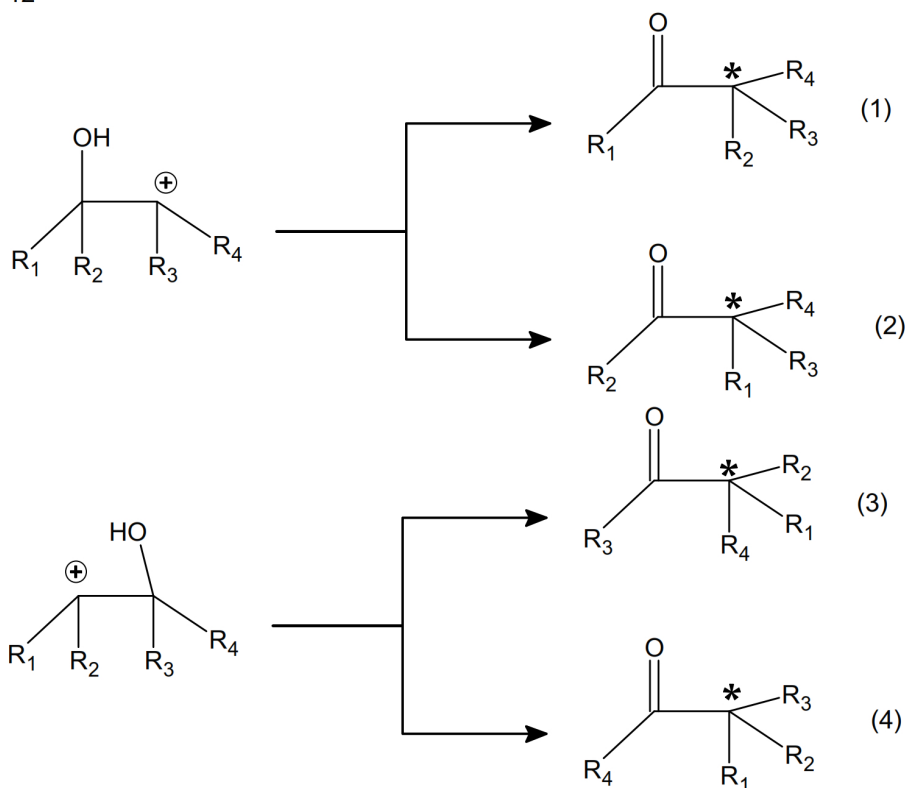


49. 80

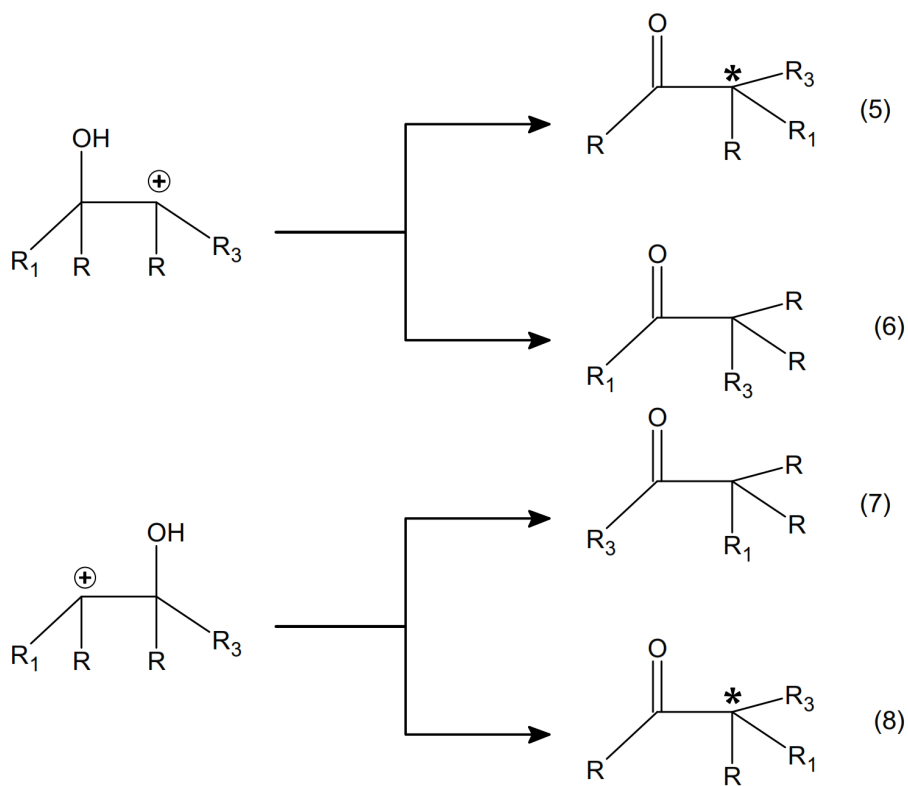
Sol. Let fraction of (+) isomer is x
 $32 = 40 \times x + (1 - x)(-40)$
 $32 = 80x - 40$
 $x = 0.9$
 + isomer is 90%
 - isomer is 10%
 Optical purity = $90 - 10 = 80\%$

50. 12

Sol.



If $R_2 = R_4 \neq R_1 \neq R_3$



Structure (5) and (8) are identical.

SECTION – A

51. C

Sol. $\because M^2 = M.M = MNM = MN = M$
 $\text{sim } N^2 = N$
 $\therefore M = M^2 = M^3 = \dots$ and $N = N^2 = \dots$
 $(M^{2024} + N^{2024})^{2025} = (M + N)^{2025}$
 Now, $(M + N)^2 = M^2 + N^2 + MN + NM = 2(M + N)$
 $(M + N)^3 = 2(M + N)(M + N) = 2^2(M + N)$
 $(M + N)^{2025} = \dots = 2^{2024}(M + N)$

52. C

Sol. $\Rightarrow (x - 1)^{60} = a_0x^{60} + a_1x^{59} + \dots + a_{30}x^{30} + a_{31}x^{29} + \dots + a_{60}$
 $\therefore a_0 = {}^{60}C_0, a_1 = -{}^{60}C_1, a_2 = {}^{60}C_2, \dots$
 $\therefore k = a_0 + a_1 + \dots + a_{60} = 0$
 $k - a_{30} = -{}^{60}C_{30}$

53. B

Sol. We have
 In S_1 unit place can have 1 or 3 or 5
 In S_2 it is ${}^5C_3 \times 3 \times \frac{4!}{2!} + {}^5C_2(9 \times 2 + 6) = 600$

54. C

Sol. Centre of circle is (2, 0) lie on line $z(1 - i) + \bar{z}(1 + i) = 4$
 Hence intersection points is two.

55. A

Sol. $T_{(\theta)} = \frac{1}{2}(1 + \cos(60 - 2\theta)) + \left(\sin^2 \theta - \frac{3}{4}\right) + \frac{1}{2}(1 + \cos(60^\circ + 2\theta))$
 $= 1 + \frac{1}{2}(2\cos^\circ \cos 2\theta) + \frac{1}{4}(-1 - 2\cos \theta^\circ) = \frac{3}{4}$
 $4 \sum_{\theta=1}^{30} T_{(\theta)} = 4 \sum_{\theta=1}^{30} \frac{3\theta}{4} = 3 \sum_{\theta=1}^{30} \theta = 1395$

56. B

Sol. Let the G.P. of three unequal numbers be given by a, ar, ar², $r \neq 1$ hence $r > 1$ for increasing G.P.
 $ar^2 \leq 100 \Rightarrow a \leq \frac{100}{r^2}$

The number of geometric progression = $\sum_{r=2}^{100} \left[\frac{100}{r^2} \right]$ where [.] is G.I.F.
 $= \left[\frac{100}{4} \right] + \left[\frac{100}{9} \right] + \left[\frac{100}{16} \right] + \left[\frac{100}{25} \right] + \left[\frac{100}{36} \right] + \left[\frac{100}{49} \right] + \left[\frac{100}{64} \right] + \left[\frac{100}{81} \right] + \left[\frac{100}{100} \right]$
 $= 25 + 11 + 6 + 4 + 2 + 2 + 1 + 1 + 1 = 53$

57. A

Sol. $\lim_{x \rightarrow 0} \frac{2(a^x - 1 + b^x - 1)}{x} = e^{\ln ab} \Rightarrow ab = 6$
 $(a, b) = (1, 6), (6, 1), (2, 3), (3, 2)$
 $P(E) = \frac{4}{36} = \frac{1}{9}$

58.

D

Sol. $f'(x) = ax(x-1) \Rightarrow f'(2) = 6 \Rightarrow a = 3$

$$f'(x) = 3(x^2 - x) \Rightarrow f(x) = x^3 - \frac{3x^2}{2} + c$$

$$\therefore f(2) = 2 \Rightarrow c = 0$$

$$\therefore f(x) = x^2 \left(x - \frac{3}{2} \right)$$

59.

D

Sol. $\log_a b = \log_a c \Rightarrow a = b^{1/b} = c^{1/c}$
from curve $y = x^{1/x}$, $x > 0$ we find $1 < a < e^{1/e}$.

60.

B

Sol. $(2xydx + x^2dy) + x^2ydx + \left(\frac{y^3}{3}dx + y^2dy \right) = 0$

Put $x^2y = t$ and $\frac{y^3}{3} = u$

$$(dt + tdx) + (udx + du) = 0$$

$$\int \frac{dt + du}{t + u} = -\int dx$$

$$\Rightarrow \ln(t + u) = -x + c$$

$$\Rightarrow x^2y + \frac{y^3}{3} = k'e^{-x} \text{ at } x = 1, y = 1, k' = \frac{4e}{3}$$

Put $x = 0$, $\frac{y^3}{3} = \frac{4e}{3} \Rightarrow k = 4$

61.

C

Sol. $\int_{\alpha}^{\beta} f(x)dt + \int_{\alpha}^{\beta} f^{-1}(x)dx = 13$

$$\Rightarrow \beta^2 - \alpha^2 = 13 \Rightarrow \beta = 7, \alpha = 6$$

62.

A

Sol. Let point of intersection (h, k)

$$\Rightarrow \frac{h}{a} + \frac{k}{b} = 1 \text{ and } ah + kb = 1, \frac{a}{b} + \frac{b}{a} = 1$$

$$\left(\frac{h}{a} + \frac{k}{b} \right) (ah + kb) = 1$$

$$h^2 + k^2 + hk \left(\frac{b}{a} + \frac{a}{b} \right) = 1$$

63.

D

Sol. $e = \sqrt{\frac{2n+4}{n+1}}$

Put $n = 48$ then $e = \frac{10}{7}$ is a rational number

$$\frac{x^2}{49} - \frac{y^2}{51} = 1 \therefore \ell = \frac{2b^2}{a} = \frac{102}{7}$$

64.

C

Sol.

PQ will be focal chord and its mid point will be circum centre.

$$2h = a\left(t^2 + \frac{1}{t^2}\right), 2k = 2a\left(t - \frac{1}{t}\right), \text{ where } P \equiv (at^2, 2at)$$

eliminate t , we get locus

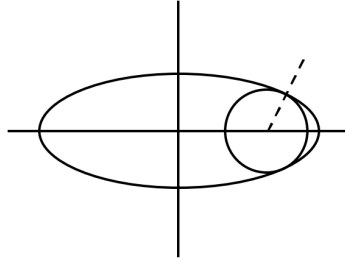
$$\text{locus is } y^2 = 2a(x - a), \text{ focus} = \left(\frac{3a}{2}, 0\right)$$

65.

C

Sol.

Centre of circle should be on major axis.

If circle touches ellipse at $P(2\cos\theta, \sin\theta)$ normal at P
 $2x\sec\theta - y\csc\theta = 3$ cuts major axis at $\left(\frac{3}{2}\cos\theta, 0\right)$ if r be the radius of circle then

$$r^2 = \left(2\cos\theta - \frac{3}{2}\cos\theta\right)^2 + (\sin\theta - 0)^2 = 1 - \frac{3}{4}\cos^2\theta$$

but $0 \leq \cos^2\theta < 1$

$$\frac{1}{2} \leq r \leq 1$$

66.

A

Sol.

Let angle between $\vec{a}$ and $\vec{b}$ is α and $\vec{a} \times \vec{b}$ and $\vec{c}$ is β

$$|(\vec{a} \times \vec{b}) \cdot \vec{c}| = 6 \Rightarrow \sin\alpha \cdot \cos\beta = 1$$

so that $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular

$$\therefore \text{req.} = |\vec{a} \times \vec{c}|^2 |\vec{d}|^2 = 9$$

67.

C

Sol.

$$\vec{a} = \vec{b} \times \vec{c} + 2\vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 2|\vec{b}|^2 \Rightarrow |\vec{a}||\vec{b}|\cos\theta = 2|\vec{b}|^2$$

$$\cos\theta = \frac{4}{|\vec{a}|} \Rightarrow |\vec{a}| = 4 \Rightarrow \theta = 0^\circ$$

$$\Rightarrow \vec{a} = 2\vec{b}$$

$$\text{Now } \vec{b} \times \vec{c} = 0 \Rightarrow \vec{b} = \vec{c} \text{ or } \vec{b} = -\vec{c}$$

$$\therefore |2\vec{a} + \vec{b} + \vec{c}| \text{ is either } |3\vec{a}| \text{ or } |2\vec{a}|$$

$$|2\vec{a} + \vec{b} + \vec{c}| \text{ is either } 12 \text{ or } 8$$

$$\text{sum} = 12 + 8 = 20$$

68. C

Sol. Normal vector of plane $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ 1 & 2 & -1 \end{vmatrix}$

$$\vec{n} = 2(\hat{i} + \hat{j} + 3\hat{k})$$

Equation of plane : $x + y + 3z = 1$

$$\therefore a + b + c = 1 + 1 + 3 = 5$$

69. B

Sol. We write the elements $A + A$

$$1 + 1, 1 + a_1, 1 + a_2, \dots, 1 + a_{18}, 1 + 77, a_1 + 77, \dots, a_{18} + 77, 77 + 77$$

It means all other sums are already present in these 39 values, which is only possible in case when all numbers are in A.P. let common diff. be 'd'

$$77 = 1 + 19d \therefore d = 4$$

$$\sum_{i=1}^{18} a_i = \frac{18}{2} [2a_1 + 17d] = 702$$

70. D

Sol.

$x_i(\text{observation})$	0	2	2^2	2^n
$f_i(\text{frequency})$	nC_0	nC_1	nC_2	nC_n

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{0 \times {}^nC_0 + 2 \times {}^nC_1 + 2^2 \times {}^nC_2 + \dots + 2^n \times {}^nC_n}{{}^nC_0 + {}^nC_1 + \dots + {}^nC_n}$$

$$= \frac{3^n - 1}{2^n} = \frac{728}{2^n} \text{ (Given)}$$

$$\Rightarrow n = 6.$$

SECTION – B

71. 30

Sol. $C_1 \equiv (1, 1), r_1 = 1, C_2 \equiv (9, 6), r_2 = 2$

$$C_1 M_1 \geq r_1, C_2 M_2 \geq r_2$$

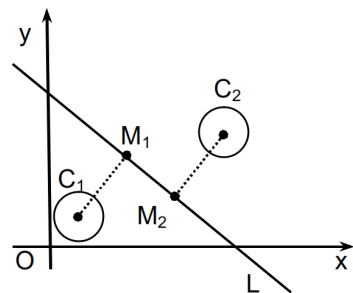
$$|7 - k| \geq 5$$

$$k \geq 12$$

$$\text{and also } k \leq 41$$

$$k \in [12, 41]$$

$$\text{no of integral } k = 30.$$



72. 41

Sol. Let $P(E_1) = x, P(E_2) = y, P(E_3) = z$

$$\Rightarrow 3x(1-y)(1-z) = (1-x)y(1-z) = 9(1-x)(1-y)z = 3(1-x)(1-y)(1-z)$$

$$\frac{3x}{1-x} = \frac{y}{1-y} = \frac{9z}{1-z} = 3$$

$$\therefore x = \frac{1}{2}, y = \frac{3}{4}, z = \frac{1}{4}$$

$$\text{Now } \begin{vmatrix} 1/2 & 3/4 & 1/4 \\ 3/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{vmatrix} = -\frac{9}{32}$$

$$\therefore \frac{a}{b} = \frac{9}{32}$$

$$\therefore a + b = 41$$

73. 8

Sol. $(2\lambda - 1 - a)2 + (3\lambda - 1)3 + (-\lambda - 1)(-1) = 0$

$$\Rightarrow 7\lambda - 2 - a = 0 \quad \dots (i)$$

And $(5\lambda - 1)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

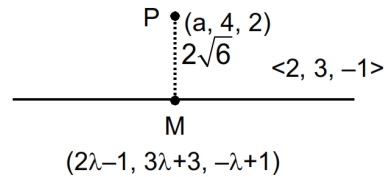
$$\Rightarrow \lambda = 1$$

$$\Rightarrow a = 5 \text{ from (i)}$$

$(\alpha_1, \alpha_2, \alpha_3)$ is reflection of P

$$\Rightarrow \alpha_1 = -3, \alpha_2 = 8, \alpha_3 = -2$$

$$\therefore a + \alpha_1 + \alpha_2 + \alpha_3 = 8.$$



74. 9

Sol. Differentiate w.r.t. x

$$f'(x) - \sin 2x = 0 - f(x) \tan x$$

or $y' + y \tan x = \sin 2x$, where $y = f(x)$

LDE, solution is $y \sec x = \int 2 \sin x \, dx$

or $y \sec x = -2 \cos x + c$

$$f(0) = 1 \Rightarrow c = 3 \therefore f(x) = 3 \cos x - 2 \cos^2 x$$

$$f(x) = -2 \left(\cos x - \frac{3}{4} \right)^2 + \frac{9}{8}$$

$$f(x) \text{ max} = 9/8.$$

75. 5

Sol. eccentricity of ellipse = $\frac{1}{2}$

$$\frac{de}{dt} = -0.1 \text{ (given)}$$

eccentricity of auxiliary circle = 0

$$\int_{1/2}^0 de = -0.1 \int_0^T dt$$

T is time at which it will become auxiliary circle

$$-\frac{1}{2} = -0.1 (T - 0)$$

$$\therefore T = 5 \text{ sec.}$$